Algorithm 3: Composite Guassian Chebyshev Quadrature(n=2,3,4)

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| Method introduction: |
| In numerical analysis Gauss-Chebyshev quadrature is an extension of Gaussian quadrature method for approximating the value of integrals of the following kind:    and    In the first case    where    and the weight    In the second case    where    and the weight |
| Algorithm Design |
| step 1:  An orthogonal polynomial *pn*(*x*) for solving the weighted function of the interval [a,b] with *w*i.  step 2:  To solve the N zero point of *pn*(*x*), that is, Gauss point.  step 3:  Calculate the integral coefficient. |
| Matlab code |
| function I = GaussChebyshevInterg(fun, a, b, n)  % GaussChebyshevInterg 用Gauss-Chebyshev型求积公式求积分计算被积函数形如 f(x)/sqrt(1-x^2)在-1到1的积分  %  % Synopsis: I = GaussChebyshevInterg(fun, a, b)  %  % Input: fun = (string) 被积函数的函数名  % a,b = 积分下限和积分上限  % n = (optional) 高斯节点的个数，默认为7  %  % Output: I = 通过Gauss型求积公式求积分的近似值  if nargin < 4  n = 7;  end    if round(n) ~= n || n < 1  error('Gauss节点个数必须为正整数');  end    p = ChebyshevIter(n); %构造n阶切比雪夫多项式  A = pi/n \* ones(n,1); %计算与切比雪夫多项式的根匹配的高斯积分系数    r = roots(p); %计算切比雪夫多项式的根  xi = ( (b-a)\*r + (a+b) ) / 2; %找出切比雪夫多项式的根在[a,b]上对应的数值xi  yi = feval(fun, xi); %计算f(xi)    I = (b-a)/2 \* (A'\*yi); %I = sum(Ai\*yi),前面的系数是积分区间改变是增加的常数  function p = ChebyshevIter(n)  % ChebyshevIter 用递推的方法计算n次勒让德-切比雪夫多项式的系数向量 Tn+2(x) = 2\*x\*Tn+1(x) - Tn(x)  %  % Synopsis: p = ChebyshevIter(n)  %  % Input: n = 勒让德-切比雪夫多项式的次数  %  % Output: p = n次勒让德-切比雪夫多项式的系数向量  if round(n) ~= n || n < 0  error('n必须是一个非负整数');  end    if n == 0 %T0(x) = 1  p = 1;  return;  elseif n == 1 %T1(x) = x  p = [1 0];  return;  end    pBk = 1; %初始化三项递推公式后项为T0  pMid = [1 0]; %初始化三项递推公式中项为T1  for i = 0:n-2  pMidCal = zeros(1,i+3); %构造用于计算的x\*Tn+1  pMidCal(1:i+2) = pMid;  pBkCal = zeros(1,i+3); %构造用于计算的Pn  pBkCal(3:i+3) = pBk;    pFwd = 2\*pMidCal - pBkCal; %勒让德-切比雪夫多项式三项递推公式Tn+2(x) = 2\*x\*Tn+1(x) - Tn(x)    pBk = pMid; %把中项变为后项进行下次迭代  pMid = pFwd; %把前项变为中项进行下次迭代  end    p = pFwd; |
| Examples and Result |
| Use Composite Guassian Chebyshev Quadrature to calculate following integral .  N=2: -0.127707018438609  N=3: -0.056452698177714  N=4: -0.037154278943705  Remarks |
| 此处写该方法程序设计的一些注意事项，也可以空白 |
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